Idempotency and the triangular inequality: some consequences of McCarthy's categoricity generalization

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G is idempotent provided it satisfies this implication [Prince and Tesar 2004]

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- Examples:
  - an idempotent grammar:

  - a *non* idempotent grammar:

- The latter example generalizes: not idempotent = chain shifts
- Idempotency is an attempt at defining a subset of opaque processes in a rule-independent way compatible with constraint-based phonology
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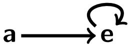
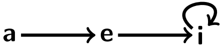
  - an idempotent grammar: 

  - a *non* idempotent grammar: 

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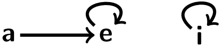
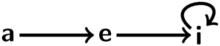
  - an idempotent grammar: 

  - a *non* idempotent grammar: 

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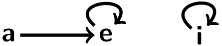
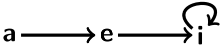
  - an idempotent grammar: 

  - a *non* idempotent grammar: 

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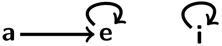
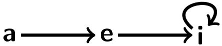
  - an idempotent grammar: 

  - a *non* idempotent grammar: 

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When does idempotency hold?

- Which conditions on the constraints guarantee that OT or HG grammars are idempotent? And what do these conditions "mean"?
- **Disclaimer**: presentation simplified by omitting conditions on correspondence relations, almost completely ignored here [Magri 2015b]
- Constraint conditions for idempotency are interesting for phonology:
  - want to model chain shifts in constraint-based phonology
  - just look up a constraint from the list of those which fail the conditions
- Constraint conditions for idempotency are interesting for learnability:
  - want to avoid chain shifts for the learner to soundly assume faithful URs for phonotactically licit training SR [Hayes 2004; Prince and Tesar 2004]
  - just make sure all constraints in your simulations belong to the list of constraints which satisfy the conditions for idempotency
- Can phonology and learnability be reconciled? Future development:
  - the learner is fine with the typology containing a chain shift **a** $\rightarrow$ **e** $\rightarrow$ **i**
  - provided the typology contains another grammar which is idempotent and phonotactically equivalent (**a** illicit; **e**, **i** licit)
  - can we use the constraint conditions for idempotency to show that attested chain shifts have this property [Moreton and Smolensky 2002]

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$$\mathbf{a} \longrightarrow \mathbf{e}$$

- Reasoning by contradiction:
  - suppose some UR is mapped to **[e]**, say **/a/**

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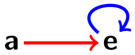

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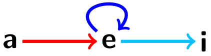

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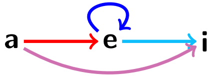

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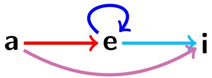

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- To get the contradiction, it is intuitively sufficient that each constraint C satisfies the following implication:

  if C prefers (**/e/**, **[i]**) to (**/e/**, **[e]**) or doesn't care
  then C prefers (**/a/**, **[i]**) to (**/a/**, **[e]**) or doesn't care

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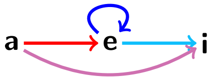

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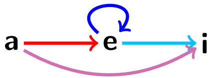

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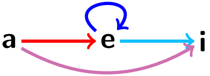

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$$\textbf{if} \quad F(\textbf{/e/}, \textbf{[i]}) = 0$$
$$\textbf{then} \quad F(\textbf{/a/}, \textbf{[i]}) \leq F(\textbf{/a/}, \textbf{[e]})$$

Idempotency in OT

OT faithfulness idempotency condition (OT-FIC)

if: $F(\mathbf{b}, \mathbf{c}) = 0$

then: $F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b})$

- If every faithfulness constraint F in the constraint set satisfies the OT-FIC, every grammar in the OT typology is idempotent [Magri 2015b; see also Moreton and Smolensky 2002; Tesar 2013; Buccola 2013]
- This is a condition which only looks at the faithfulness constraints, not at the markedness constraints
- We can go through the list of faithfulness constraints in Correspondence Theory (and its developments) and established when they satisfy the OT-FIC [Magri 2015a]

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if: $F(\mathbf{b}, \mathbf{c}) = 0 + \xi$

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for every threshold $\xi \geq 0$

- If every faithfulness constraint F in the constraint set satisfies the HG-FIC, every grammar in the HG typology is idempotent
- Sanity check:
  - HG typologies are larger than OT typologies
  - a stronger condition is needed to discipline all HG grammars to comply
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What do these FICs conditions mean? Can they be interpreted?

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Faithfulness triangular inequality

- Faithfulness constraints intuitively measure the phonological distance between underlying and surface representations

- Do faithfulness constraints satisfy the various conditions which pertain to the axiomatic definition of distance or metric? [Rudin 1953]

- One crucial metrical axiom is the triangular inequality:
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  - $dist(\mathbf{a},\mathbf{c}) \leq dist(\mathbf{a},\mathbf{b}) + dist(\mathbf{b},\mathbf{c})$

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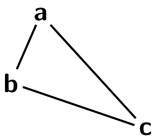

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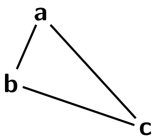
- Faithfulness constraints intuitively measure the phonological distance between underlying and surface representations

- Do faithfulness constraints satisfy the various conditions which pertain to the axiomatic definition of distance or metric? [Rudin 1953]

- One crucial metrical axiom is the triangular inequality:
  - the side of any triangle is shorter than the sum of the other two sides



  - $dist(\mathbf{a}, \mathbf{c}) \leq dist(\mathbf{a}, \mathbf{b}) + dist(\mathbf{b}, \mathbf{c})$

Faithfulness triangular inequality (FTI)

$F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b}) + F(\mathbf{b}, \mathbf{c})$

Metric interpretation of the HG-FIC

First point made by this talk

For an arbitrary faithfulness constraint F:

HG-FIC		FTI
if: $F(\mathbf{b},\mathbf{c}) \leq \xi$ **then:** $F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b}) + \xi$	$\Longleftrightarrow$	$F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b}) + F(\mathbf{b},\mathbf{c})$

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- This equivalence holds because:
  - assume that $\xi = F(\mathbf{b},\mathbf{c})$
  - then the FTI is analogous to the consequent of the HG-FIC
- This equivalence means that:
  - the HG-FIC simply requires a faithfulness constraint to measure phonological distance in compliance with the triangular inequality
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Towards a metric interpretation of the OT-FIC

Second point made by this talk: preliminary formulation

For every binary faithfulness constraint F (which take values 0 or 1):

OT-FIC	$\Longleftrightarrow$	**FTI**
if: $F(\mathbf{b},\mathbf{c}) = 0$ **then:** $F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b})$		$F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b}) + F(\mathbf{b},\mathbf{c})$

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- Why this equivalence holds:
  - If the antecedent of the OT-FIC holds:
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  - If the antecedent of the OT-FIC fails:
    $\Longrightarrow$ that makes the right-hand side of the FTI large enough
- The FTI entails the OT-FIC independently of binarity
  but the equivalence fails for non-binary faithfulness constraints
- This makes sense: FTI = HG-FIC > OT-FIC
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Categoricity: the idea

McCarthy's categoricity conjecture [McCarthy 2003]

Each faithfulness constraint F useful in phonology is categorical

- Intuitively, $\text{IDENT}_{[\text{NASAL}]}$ is categorical because:
  - $\text{IDENT}\begin{pmatrix} \text{n t k} \\ |\ /\ / \\ \text{t ŋ} \end{pmatrix} = \text{IDENT}\begin{pmatrix} \text{n t k} \\ | \\ \text{t ŋ} \end{pmatrix} + \text{IDENT}\begin{pmatrix} \text{n t k} \\ / \\ \text{t ŋ} \end{pmatrix} + \text{IDENT}\begin{pmatrix} \text{n t k} \\ / \\ \text{t ŋ} \end{pmatrix}$
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- In general, categoricity means that a phonological candidate can be broken up into "sub-candidates" in such a way that:
  - $F(\text{cand}) = \sum_{\text{sub-cand}} F(\text{sub-cand})$ — the violations assigned by F to the candidate is the sum of the violations it assigns to the "sub-candidates"
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- Categoricity is intimately related to monotonicity

- Intuitively, $\textsc{Ident}_{[\textsc{nasal}]}$ is monotone because violations increase when candidates increase through additional correspondence relations:

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Categoricity and monotonicity: more details

- Candidate = UR + SR + correspondence [McCarthy and Prince 1995]
- A candidate can be split into sub-candidates along any of these three dimensions, yielding three notions of categoricity and monotonicity
- Faithfulness categoricity:
  - C-categoricity: sub-candidates have one (few) corresponding pair (IDENT)
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Categoricity+monotonicity in natural language phonology

Extended categoricity conjecture

Any faithfulness constraint F relevant for Natural Language is

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  - INTEGRITY, UNIFORMITY
  - IDENT_{φ}
  - disjunction and conjunction [Smolensky 1995; Downing 2000]
  - LINEARITY, MAXLINEARITY, DEPLINEARITY [Heinz 2005]
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Metric interpretation of the OT-FIC

Second point made by this talk

For every F which satisfies the extended categoricity conjecture:

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HG idempotency

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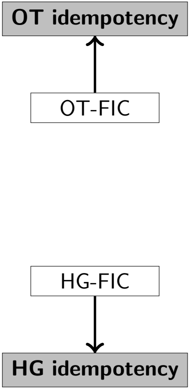

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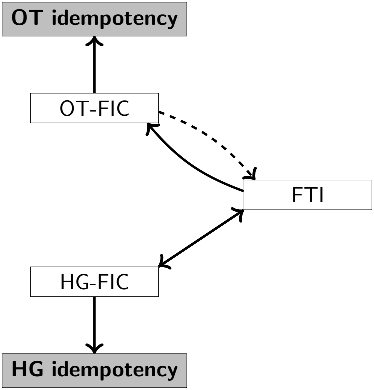

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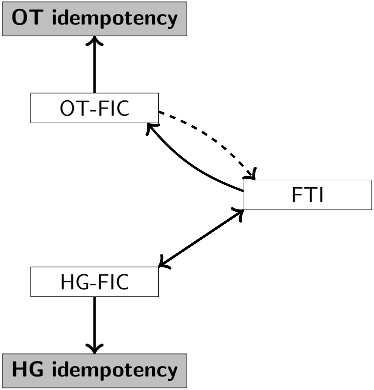

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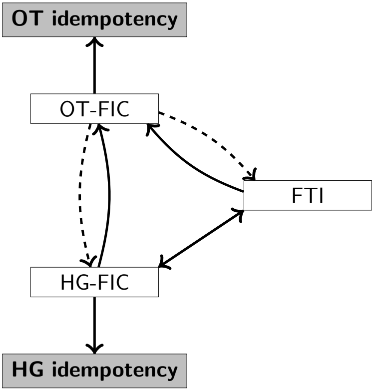
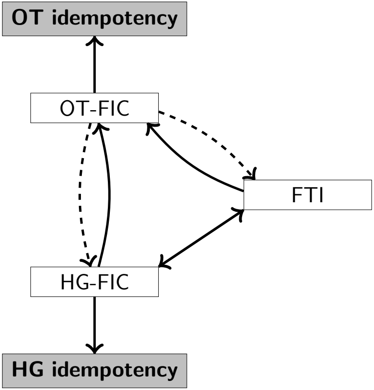


- Idempotency is related to the metrical nature of faithfulness:
  - HG: the relation holds unrestricted
  - OT: it requires categoricity
- A non-trivial implication of McCarthy's categoricity conjecture
- Given categoricity, idempotency in HG does not require additional constraint conditions than in OT
- The constraints which satisfy the OT-FIC, also satisfy the HG-FIC (and the FTI)

Conclusions



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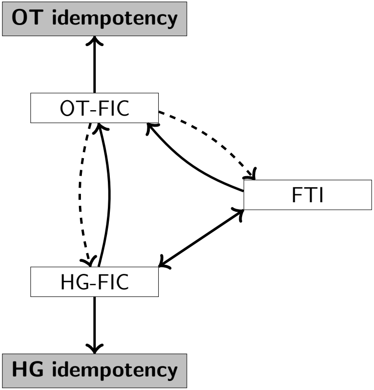

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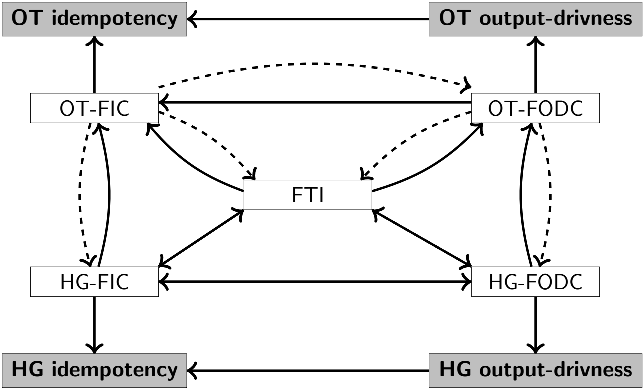

Thank you!

[Slides available on my website, together with the two papers that this talk is based on: Magri (2015a) and Magri (2015b)]

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