Introduction. In standard lattice-theoretic approaches to natural language (e.g., Link 1983, Landman 2000, Champollion 2017) singularities and pluralities are assumed to involve two distinct mereological structures and it is supposed that compositional semantics is not sensitive to the manner how parts of a whole are arranged. In this paper, I argue that: i) some quantificational operations including counting presuppose certain topological relations to hold within a part-whole structure and ii) quantification over parts and wholes are subject to identical restrictions. I present new evidence in favor of a mereotopological approach to nominal semantics (Grimm 2012) and novel data concerning the interaction between quantification and part-whole relations.

Puzzle. Moltmann (1997) observes an analogy between partitives involving proportional quantifiers and singular and plural terms which suggests a unified parthood structure for both singular and plural entities, see (1) (the pattern holds cross-linguistically). Moreover, partitives with number-neutral nominals such as object mass nouns and pluralia tantum are ambiguous between a part-of-a-singularity and part-of-a-plurality reading, see (2). A similar effect is reported in languages with general number, see (3) (Sauerland & Yatsushiro 2004).

(1) Teil des Apfels/der Äpfel
part of-the apple/of-the apples
German
(2) połowa obuwia/nożyczek
half of-the-footwear/scissors
Polish
(3) John-wa hotondo hon-o yomi-oeta
John-TOP most book-ACC read-finished
‘John finished reading most of the books/most parts of the book(s).’
Japanese

However, the fact that part words modifying plurals are uncountable (on a part-of-a-plurality reading) has been claimed to be a counterargument for a unified mereology (Schwarzschild 1996). While (4a) denotes 3 subdivisions of the wall, (4b) cannot refer to 3 subsets of the walls. I argue that the phenomenon results from the fact that regular plurals refer to scattered entities. Crucially, since Italian irregular plurals employ the notion of integrity or cohesion of a sum (Ojeda 1995, Acquaviva 2008), counting parts of a plurality is valid, see (4c).

(4) a. tre parti del muro b. #tre parti dei muri c. tre parti delle mura
three parts-of-the wall three parts-of-the walls three parts-of-the walls
Italian

Proposal. I model singular individuals in terms of mereotopology where topological relations between parts such as connectedness are specified, whereas pluralities are modeled in terms of mereology, and thus bear not topological commitments. Specifically, count singulars incorporate the notion of maximally strongly self-connected (MSSC; Casati & Varzi 1999) which guarantees that an entity is an integrated whole, see (5). On the other hand, plurals denote arbitrary sums of MSSC entities, see (6), whereas Italian irregular plurals refer to clusters (Grimm 2012), i.e., pluralities formed by connected singular parts, see (7). To account for the partitive constraint (de Hoop 1997), I assume that definite articles introduce the standard maximization operator. A part word is modeled in terms of proper parthood (Barker 1998). In addition, I posit a partitioning function π which selects a set of entities P and yields its subset π(P) such that it is a set of those elements in P that do not overlap. Application of MSSC to π(P) would then yield a contiguous part forming an integrated object, see (10). Finally, I assume that counting is mapping of entities to numbers via the measure function # (Krifka 1989) and argue that it is only possible to count entities or parts that are conceptualized as integrated objects, see (11). The LF structure in (12) explains the data in (4) since arbitrary sums are not integrated objects, i.e., cannot be counted. Thus, singulars and plurals share a unified part-whole structure but differ wrt topology.
(5) \[ \text{[muro]} = \lambda x [\text{MSSC(WALL)}(x)] \]
(6) \[ \text{[muri]} = \lambda x [^* \text{[muro]}(x)] \]
(7) \[ \text{[mura]} = \lambda x [\text{CLUSTER}([\text{muro}](x))] \]
(8) \[ \text{[DEF]} = \lambda P \text{[MAX}(P)] \]
(9) \[ \text{[PART]} = \lambda y \lambda x [x \sqsupset y] \]
(10) \[ \text{[IND]} = \lambda P \lambda x [\text{MSSC}(\pi(P))(x)] \]
(11) \[ \text{[tre]} = \lambda P. P_{\text{MSSC}} \lambda x [^* P(x) \land \#(P)(x) = 3] \]
(12) \[ \text{[Numeral [IND [PART [DEF NP]]]]} \]

References. Acquaviva (2008) 
Lexical plurals • Barker (1998) 
Partitives, double genitives and anti-uniqueness • Champollion (2017) 
Parts of a whole • Casati & Varzi (1999) 
Parts and places • de Hoop (1997) A semantic reanalysis of the partitive constraint • Grimm 
(2012) Number and individuation • Ionin et al. (2006) 
Parts of speech • Krifka (1989) 
Nominal reference, temporal constitution and quantification in event semantics • Landman (2000) 
Events and plurality • Link (1983) The logical analysis of plural and mass nouns • 
Pluralities